spectra, and different curves were obtained for the K$\alpha$ (333) and K$\beta$ (333) spectra.

*Graphite.*—Only for the cleavage plane reflection was it possible to obtain a definite temperature-intensity curve, and for the direction perpendicular to this plane an unusually high coefficient of expansion was measured.

*Diamond.*—No decrease in intensity was found that could be measured with certainty, and a very small thermal agitation would be expected on account of the diamond structure's great strength.

*Ruby and Sapphire.*—An anomalous effect was observed, since the decrease of intensity of the (111) spectra was greater than that of the (222). This may be completely explained by assuming that the atoms of the aluminium pair remain in contact and do not share in the expansion of the lattice.

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# *An Investigation of the Colour Vision of* 527 *Students by the Rayleigh Test.*

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Lord Rayleigh discovered, in 1881,* that if homogeneous yellow is matched with a mixture of homogeneous red and homogeneous green, some persons require much more red, others much more green, in the mixture than the normal. The former have been called red anomalies, the latter green anomalies, and both together have been grouped under the name of anomalous trichromats, because, according to Rayleigh, they are generally not dichromats. Rayleigh obtained matches for 23 male observers. Of these 16 agreed with himself within the errors of observation, and were regarded as normal. Five of the remainder were green anomalies and two red anomalies. Among seven female observers there was not one whose colour vision differed sensibly from his own. Rayleigh did not publish his data, but states that one of the green anomalies had the ratio green to red 2·09 times as great as himself, and one of the red anomalies had the ratio red to green 2·6 times as great as himself. He states that, although the number of observers is insufficient for statistical purposes, it is evident that the peculiarity is by no means rare, at least, among men; he also states that it would seem as if normal colour vision were not of the nature of an average from which small deviations are more probable than larger ones, but that this would require confirmation.

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* 'Nature,' vol. 25, p. 65 (1881).

In 1890, Schuster* published an account of a more extensive investigation carried out with the same apparatus; 75 individuals were examined; of these three proved colour blind; four, of whom three belonged to the same family, were green anomalies, while one was a red anomaly. Schuster states: (i) that there cannot be any doubt of the real existence of small differences following the ordinary law for deviation from a mean; and (ii) that the larger differences seem certainly more frequent than the distribution of small differences would lead us to expect.

Other investigations on the subject have been carried out by Von Maltzew,† Gutmann,‡ and Edridge-Green.§ Von Maltzew examined 17 cases who were obviously not a random distribution. Gutmann, unfortunately, gives no data, and always takes for granted the existence of two well defined classes of anomalous trichromats, the green and red anomalies, so his work is useless from a statistical point of view. Edridge-Green examined 100 women students, and found that 86 were normal, 10 red anomalies, two green anomalies, while two occupied intermediate positions, but he also does not publish individual readings.

The question of the existence of the green and red anomalies as well defined classes, and their relation to the colour blind, is therefore by no means settled yet, and it was thought desirable, as part of the survey of the colour vision of students which is at present being carried on at Glasgow, to devote special attention to this question. Consequently, during the past year, the colour vision of 527 students has been tested by an apparatus similar to Lord Rayleigh's, and the present paper describes the results.

The apparatus consisted of a spectrometer fitted with a single dense flint prism. The latter was set at minimum deviation for Na light. Between the prism and the telescope object-glass was set up a vertical glass plate, on which was cemented a Wollaston double-refracting prism made in Iceland spar; this glass plate was held in a grating holder. The light which passed through the Wollaston prism was decomposed into two beams polarised respectively in the horizontal and vertical planes; these two beams at the same time suffered deviations, one towards the base of the prism and the other away from it. The light which did not pass through the Wollaston prism, but only through the glass plate to which the latter was cemented, arrived at the object-glass of the telescope without further deviation.

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* 'Roy. Soc. Proc.,' vol. 48, p. 140 (1890).

† 'Zs. f. Sinnesphysiol.,' vol. 43, p. 76 (1908–9).

‡ *Ibid.*, vol. 42, pp. 24, 250 (1907); vol. 43, pp. 146, 199, 255 (1908–9).

§ 'Roy. Soc. Proc.,' B, vol. 86, p. 164 (1913).

In the focal plane of the telescope there was an adjustable slit, and behind this an extremely good nicol eyepiece, the position of which could be read on a divided circle. Both collimator slit and eyepiece slit were kept at a width of $\frac{1}{2}$ a millimetre, and the telescope adjusted, so that the Na image formed by the light which missed the Wollaston prism fell exactly on the eyepiece slit. Superimposed on this image were the two formed by the beams which passed through the Wollaston prism. One of these was exactly the colour of Tl green (5351 A.U.); the other was a red of wave-length 6300 A.U. When the eyepiece slit was viewed through the eyepiece, it was seen illuminated by red, yellow, and green light superimposed. If, however, the eyepiece was removed, and the observer looked through the nicol and slit alone at the prism, he saw the face of the prism in Na yellow and superimposed on it the diamond-shaped Wollaston prism, the colour of the latter changing from red to green, through yellow, as the nicol was rotated. As viewed through the slit the face of the flint prism appeared almost square, each side subtending an angle of 6°; the diamond appeared in the middle of the square, and covered about one-third of its area.

The students tested comprised all those taking a course in the optical laboratory during the three terms of Session 1921–22, with one or two exceptions, owing to absence and some additions; 423 were men and 104 women. They were essentially a random distribution. They were taken in pairs from the experiment on which they were employed, optical bench, or whatever it was, and no one who was asked to make the test declined. Each made two settings, one in each of two adjacent quadrants; suppose, for example, these settings were 26·0° and 127·2°, which are very near the mean values, then the square of the cotangent of half the difference, $\cot^2\frac{1}{2}(127{\cdot}2-26{\cdot}0) = 0{\cdot}6748$, gave the ratio of the intensity of red to intensity of green in the mixture, the intensities of red and green being measured in different units.

Certain precautions have to be observed in making the tests. First of all, at the wave-length selected for the yellow, Na yellow, the colour changes very rapidly with the wave-length, and a slight difference in the tint of the yellow means a large difference in the ratio of red to green. Owing to students seizing the telescope too roughly, these slight differences occurred occasionally, but they were always detected, as I read the instrument after every two students. Then the lamp used as source, a 40-watt Osram, must burn at the same colour throughout the tests. Before the investigation was started, three lamps of exactly the same colour were selected out of a batch of twenty; the first burned out and was replaced by the second half-way through the investigation. For the colour to be the same, the voltage must be constant; as the lamp was simply connected to the town leads the voltage was subject to small variations. I stopped the tests on two occasions, as it appeared the readings were changing appreciably owing to a slight change of voltage, but I could not be sure whether the change was a real one or not; however, it was thought better to err on the side of excessive caution.

If immediately before the test the eye is fatigued with red or green, a different setting is obtained. My usual value for the ratio of red to green is about 0·588. On fatiguing my eyes by gazing at a strong red light for 7 minutes and making a setting as rapidly as possible thereafter, I obtained 0·840, but the eye recovered fully in about 1 minute. On another occasion, on fatiguing my eyes in the same way, and reading as rapidly as possible, I obtained 0·729 instead of 0·588. This time my eyes took 2 minutes to recover. The room in which the tests were made was, however, darkened, and of no pronounced colour, so there was no occasion to suspect that the results were affected by fatigue.

Some of the 423 men were colour blind. These were asked to make a setting the same as the others. In every case they had a preference for one match, though in one or two cases they stated it was the best of a bad business, and one man required to be assisted by leading questions: it was, however, lack of self-confidence he was afflicted with rather than colour blindness. As it is very difficult to draw a line between the colour blind and the others, they have been entered in the Table and diagram along with the others.

Hitherto, it has been usual to state the results in terms of intensity of red light to intensity of green in the mixture, or *vice versa.* There is no particular reason why the results should be stated in terms of red to green rather than of green to red; both red and green enter into the mixture in precisely the same manner. The shape of the curve differs considerably according as red or green is in the numerator. I have thought it advisable, therefore, to state the results in terms of logarithm of ratio of intensity of red to green. When the logarithm of the ratio is taken as abscissa, inverting the ratio does not alter the shape of the diagram, but merely interchanges left and right.

The results are given in the following Table and in the diagram on p. 358:—

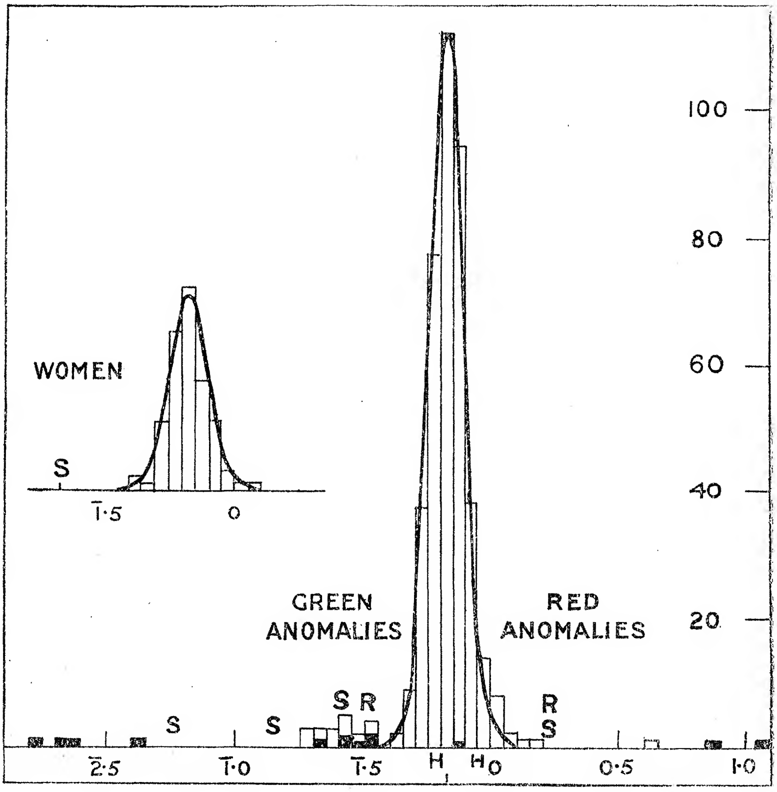
| Logarithm of ratio of red to green. | Men. | | Women. | |
|---|---|---|---|---|
| | Observed. | Calculated. | Observed. | Calculated. |
| $\bar{1}\cdot25$ and $<\bar{1}\cdot30$ | 3 | | | |
| $\bar{1}\cdot30$ $<\bar{1}\cdot35$ | 3 | | | |
| $\bar{1}\cdot35$ $<\bar{1}\cdot40$ | 3 | | | |
| $\bar{1}\cdot40$ $<\bar{1}\cdot45$ | 5 | | | |
| $\bar{1}\cdot45$ $<\bar{1}\cdot50$ | 2 | | | |
| $\bar{1}\cdot50$ $<\bar{1}\cdot55$ | 4 | | | |
| $\bar{1}\cdot55$ $<\bar{1}\cdot60$ | 0 | 0·2 | | |
| $\bar{1}\cdot60$ $<\bar{1}\cdot65$ | 2 | 1·9 | 2 | 0·5 |
| $\bar{1}\cdot65$ $<\bar{1}\cdot70$ | 9 | 11·5 | 1 | 3 |
| $\bar{1}\cdot70$ $<\bar{1}\cdot75$ | 37 | 41·8 | 11 | 11 |
| $\bar{1}\cdot75$ $<\bar{1}\cdot80$ | 77 | 87 | 25 | 23 |
| $\bar{1}\cdot80$ $<\bar{1}\cdot85$ | 112 | 112 | 32 | 29·5 |
| $\bar{1}\cdot85$ $<\bar{1}\cdot90$ | 94 | 87 | 17 | 23 |
| $\bar{1}\cdot90$ $<\bar{1}\cdot95$ | 38 | 41·8 | 11 | 11 |
| $\bar{1}\cdot95$ $<0\cdot00$ | 14 | 11·5 | 3 | 3 |
| $0\cdot00$ $<0\cdot05$ | 8 | 1·9 | 1 | 0·5 |
| $0\cdot05$ $<0\cdot10$ | 2 | 0·2 | 1 | |
| $0\cdot10$ $<0\cdot15$ | 1 | | | |
| $0\cdot15$ $<0\cdot20$ | 1 | | | |

There were also men at $\bar{3}\cdot079$, $\bar{2}\cdot230$, $\bar{2}\cdot301$, $\bar{2}\cdot370$, $\bar{2}\cdot633$, 0·624, 0·876, 1·085.

We see from the Table, for example, that 77 men obtained a ratio of red to green, the logarithm of which lay in the range $\bar{1}\cdot75$ to $\bar{1}\cdot80$, including the lower limit and excluding the upper. These are represented in the diagram by a rectangle bounded by vertical lines through $\bar{1}\cdot75$ and $\bar{1}\cdot80$, and 77 units high. In order to avoid extending the diagram too far the man at $\bar{3}\cdot079$ has not been represented.

A Gaussian curve was fitted to all the data for the women, as shown in the diagram. Its maximum is at $\bar{1}\cdot825$, and the calculated values shown in the Table are the mid-ordinates of each rectangle. The Gaussian curve shown in the diagram for the men, and the corresponding calculated values are obtained from the curve for the women by increasing all the ordinates in a given ratio; a better agreement might have been obtained by fitting a curve directly to the data for the men, but there would have been a certain amount of arbitrariness in deciding how many of the outlying observers to include.

The question arises as to how much of the "scatter" of the Gaussian curve is due to error of observation, inexperience in making the readings, accidental variations, etc., and how much is due to a real difference in the physiological equipment of the observer. Each observer made two settings which meant two independent determinations; by taking the square of the difference and summing for a large number of observers, it was possible to calculate the



"scatter" which would be obtained if all the readings were made by an average observer, and it seems that about one-half of the breadth of the Gaussian curve is due to these accidental variations. This is a much larger proportion than I expected before making the calculation. My own setting is indicated by $H_1$; by fatiguing my eyes as much as possible with red light I got the setting H. Never under ordinary circumstances do I get the reading out by more than half the breadth of a rectangle either way. But some observers are much more erratic than others.

None of the women investigated showed any abnormality of colour vision. Of the men the following ten were very colour blind: (92) $\bar{2}$·663, (137) $\bar{2}$·301, (144) $\bar{3}$·079, (145) 0·876, (149) $\bar{2}$·230, (233) 1·085, (269) $\bar{1}$·516, (350) $\bar{1}$·894, (357) $\bar{1}$·406, (358) $\bar{2}$·370; the number in the bracket is the observer's number, the other is the logarithm of his ratio. The following four were partially colour blind: (244) $\bar{1}$·349, (250) $\bar{1}$·547, (274) $\bar{1}$·433, (405) $\bar{1}$·452. The foregoing are entered in black in the diagram. The following were put through a very stringent test, as it was thought, owing to their setting, that their colour vision might be defective, but the test showed they were as good as the normal: (105) $1\cdot255$, (293) $\bar{1}\cdot403$, (339) $0\cdot624$, (342) $\bar{1}\cdot361$, (411) $\bar{1}\cdot338$, (104) $\bar{1}\cdot336$.

We come now to consider the results. There is no doubt whatever of the existence of small differences following the ordinary law of error, and it is clear that the terms green anomaly and red anomaly have hitherto been used in two senses. Edridge-Green found 12 anomalies among 100 women students. My 104 women students all group themselves according to the ordinary law of error. Edridge-Green consequently applies the term to the outlying portions of the normal curve. Schuster, on the contrary, restricts the term anomalous trichromat to the scattered observers outside the normal curve. Schuster had, altogether, 3 colour blind and 5 anomalous trichromats out of 75 observers. I have 14 colour blind and 15 others outside the normal curve out of 423 men students. The proportion agrees as well as can be expected. I have indicated the positions of Schuster's anomalous trichromats by S, and two of Rayleigh's anomalous trichromats by R in the diagrams; these positions are, however, only approximate, as Rayleigh and Schuster used a different red and green from mine.

With reference to the women, if it were not for Schuster's observer, I should have said that normal variation covered everything. There are about 28 men outside the normal curve; if the proportion were the same, there should be about 7 women; but there are none. It is clear, therefore, that normal variation almost covers everything as regards the women. As regards the men, I have not yet found a colour blind man who is a dichromat, though I have now worked through a population of 1500; trichromasy, in my experience, seems to approach monochromasy directly without passing through dichromasy as an intermediate stage.* I consequently do not like the term anomalous trichromats as a designation for those observers outside the normal curve who are not colour blind, but prefer the terms red and green anomalies. Using the terms in this sense, I have thus one or two red anomalies and 14 green anomalies. Although some of these have remarkably good colour vision, it is not easy to draw a sharp distinction between them and the colour blind; there are intermediate cases.

There is no doubt that the anomalies cannot be explained as extreme cases of normal variation. But they are not nearly so common as would be anticipated from Rayleigh's first paper. And they do not appear to form well defined classes.

Some observers see the red end of the spectrum much dimmer than others, and recently a survey has been started to see whether these are extreme cases of normal variation, or an independent group. After preliminary work with a spectrometer, wedge photometer, and flicker photometer, it was found that matching a red lamp against a white light by means of a photometer of the grease spot type gave the most satisfactory results. The red light was spectrally very pure. This survey has overlapped the one described in this paper to the extent of about 120 observers, and the following Table includes all the colour blind and red and green anomalies of the present paper who have also made the photometer test. The first column gives the observer's number, the second the logarithm of his ratio of red to green, and the third the photometer reading; a low reading means that the observer is very blind to red. The scale has not yet been calibrated, but the observer (357), at the one end, sees red about 60 times as bright as the observer (149), at the other. The normal setting on the photometer scale is about 18; the Table includes the two most red blind and two most red sensitive observers yet disclosed by the photometer survey :—

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* 'Roy. Soc. Edin. Proc.,' vol. 42, p. 75 (1922).

| | | | |
|---|---|---|---|
| (149) | 2·230 | $<0$ | colour blind. |
| (293) | $\bar{1}$·403 | 16·5 | green anomaly. |
| (334) | $\bar{1}$·376 | 15·6 | green anomaly. |
| (339) | 0·624 | 1·68 | red anomaly. |
| (342) | $\bar{1}$·361 | 12·5 | green anomaly. |
| (350) | $\bar{1}$·894 | 16·0 | colour blind. |
| (357) | $\bar{1}$·406 | 27·4 | colour blind. |
| (358) | $\bar{2}$·370 | 25·0 | colour blind. |
| (405) | $\bar{1}$·452 | 19·4 | partially colour blind. |
| (411) | $\bar{1}$·338 | 19·2 | green anomaly. |

If we examine the Table, we find one very colour blind man (350), normal in both tests; one colour blind (149), and one red anomaly (339), extremely red blind. Four green anomalies (293), (334), (342), (411), and one partially colour blind man (405), who makes the same reading as these four in the Rayleigh test, about normal in the photometer test; and two colour blind (357) and (358), extremely red sensitive. So the probability is that those who are abnormal in the Rayleigh test will be abnormal also in the photometer test, but there are marked exceptions; also, from the nature of the abnormality in the one test, it is not possible to predict the nature of the abnormality in the other.